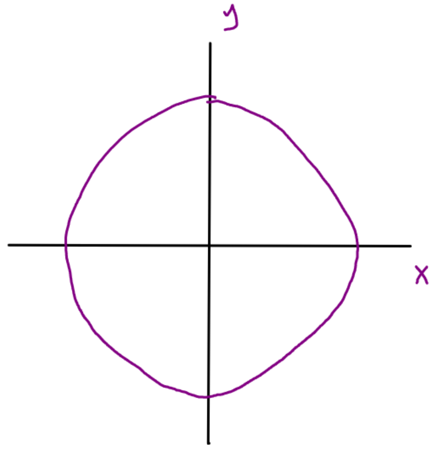
SIMPLE SURFACES IN 3-SPACE

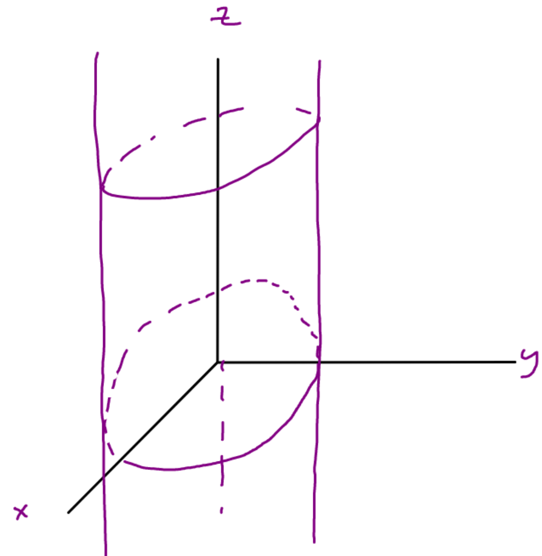
GOAL: Discuss some simple surfaces in $\mathbb{R}^3$.

I. Cylinders are surfaces obtained from plane-curve graphs.

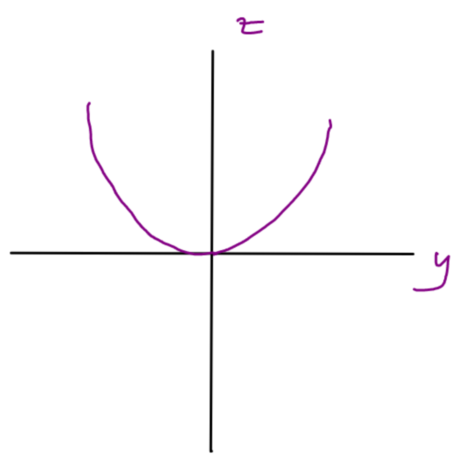
Ex: The following are cylinders.

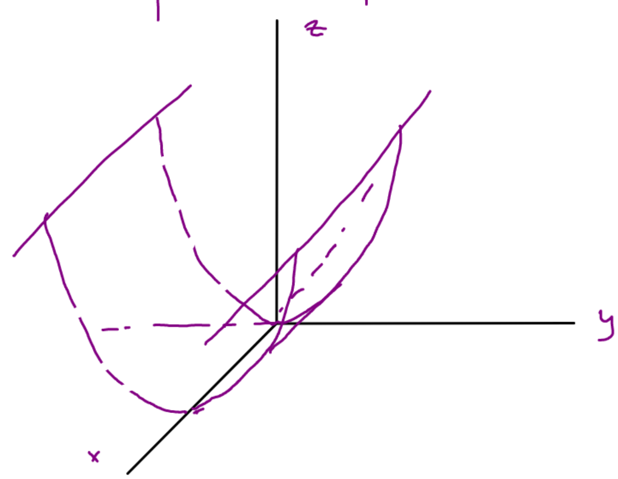
$x^2+y^2=1$





$z=y^2$





NB: Defining equations of cylinders involve two variables only... after some rotation and stretching, if need be...

II. Quadratic surfaces are determined by polynomial equations of degree 2 involving three variables.

Thus up to some algebraic manipulations (which we won't do here, because they're tedious and are somewhat beyond the scope of Calculus III at this level), every quadratic surface has an equation of one of the forms

$$Ax^2 + By^2 + Cz^2 = D \quad \text{OR} \quad Ax^2 + By^2 + Cz = 0.$$

Classifying these objects then depends on the signs of nonzero constants A, B, C, (and D). In practice we think of the constant $E = \pm \frac{1}{e^2}$ and divide by D (to use $D = 1$). After this analysis, we obtain the following table of surfaces:

NAME	EQUATION	TRACES
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	elliptical or empty
Elliptic Paraboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z}{c} = 0$	elliptical and parabolic
Hyperbolic Paraboloid	$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z}{c} = 0$	hyperbolic and parabolic
Cone	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$	(general) elliptical and hyperbolic
One-sheet Hyperboloid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$	elliptical and hyperbolic
Two-sheet Hyperboloid	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	(elliptical or empty) and hyperbolic.